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Book Review

Infinite-dimensional Systems in Atmospheric and Oceanic Science by Guo Boling, Huang Daiwen World Scientific; Zhejiang Science and Technology, 2014;
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Any problem of geophysical fluid dynamics involves a huge (theoretically infinite) number of degrees of freedom coupled via complex nonlinear interactions. So, the methods of dynamical systems theory provide very important and very powerful mathematical tools for analysis of such problems.

"Infinite-dimensional Systems in Atmospheric and Oceanic Science" underlines the key role of dynamical systems theory methods in atmospheric and oceanic investigations. It consists of five chapters.

Chapter 1, Nonlinear Equations of the Atmospheric and the Oceanic Motions, reviews the physical setting of the basic and primitive equations (PEs) of the atmosphere and the oceans. First, a standard treatment of a rotating coordinate frame is presented along with how it differs from an inertial reference frame. The basic equations of the atmosphere are formulated. These include: the atmospheric momentum equation, the continuity equation, the atmospheric state equation, and the atmospheric thermodynamic equation. Next, the fundamental equations of the oceans are listed, and the Boussinesq approximation is elucidated. In the following section the equations of the atmosphere and the oceans are deduced in spherical coordinates. The basic philosophy of the hydrostatic approximation is explained and the dry atmospheric equations in isobaric coordinates are derived. The way of representing topography in atmospheric models is discussed also, and the basic equations in the topography coordinate frame, which govern the atmospheric motion, are presented. The set of β-plane equations as well as the governing equations for the large-scale atmospheric and oceanic motions are obtained and analysed briefly. Lastly, the top and bottom boundary conditions of the atmosphere and the oceans are formulated and briefly discussed.

Chapter 2, Some Quasi-Geostrophic Models, deals with the important concept in the study of atmospheric and oceanic dynamics, namely, the geostrophic approximation. Section 2.1 discusses the barotropic model, known also as a shallow water model. Using the WKB method, the two-dimensional quasi-geostrophic equations are obtained as the rational simplification of the barotropic model. In section 2.2, the small parameter method is used to derive a simplified version of the adiabatic frictionless atmospheric equations, known as the three-dimensional quasi-geostrophic system of equations. A similar procedure is also applied to obtain the baroclinic oceanic quasi-geostrophic equation. The next section presents the formulation of a two-layer version of the quasi-geostrophic model. Finally, the Cauchy problem for the surface quasi-geostrophic equation is considered. It describes the dynamics of quasi-geostrophic flow with uniform potential vorticity, i.e., the evolution of the lower surface potential temperature (or buoyancy), on horizontal boundaries. The following problems are addressed: the global existence of weak solutions, the growth of solutions, the regularity of the solutions, the global well-

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posedness of the critical dissipative surface quasi-geostrophic equations, blow up for the solutions, etc.

Chapter 3, Well-Posedness and Global Attractors of the Primitive Equations, discusses some mathematical aspects of the PEs of the large-scale atmosphere and oceans. Section 3.1 studies the initial-boundary value problem for the PEs of the large-scale moist atmosphere. A mathematical formulation of the PEs is given and the global existence of weak solutions together with the existence of the trajectory attractors and global attractors are proved. Another section deals with the asymptotic behavior of the strong solutions of the PEs of the large-scale atmosphere in the pressure coordinates. First, the four main results about global existence and uniqueness of strong solutions of the PEs are formulated. Next, some auxiliary lemmas concerning the a priori estimates of local strong solutions are established. These results are necessary to prove the two main theorems. Then, some basic facts about the infinite-dimensional dynamical system theory are recalled, and applied to prove the other two theorems regarding the existence of global attractors of atmospheric PEs. The following section concentrates on the well-posedness of the PEs of the large-scale dry atmosphere. The main theorems are formulated and proved, namely, the global existence and uniqueness of weakly strong solution and the global existence of the smooth solutions of the initial-boundary value problem for the PEs. The last section is devoted to the concise study of the global well-posedness of the viscous PEs of the large-scale oceans.

Chapter 4, Random Dynamical Systems of Atmosphere and Ocean, focuses on the application of stochastic methods to the atmosphere and oceans. It starts with the dynamical system generated by the two-dimensional quasi-geostrophic equation with random external force and dissipation. Next, the global existence and uniqueness of solutions of the stochastic initial-boundary value problem are analysed and proved. The problem of the existence of random attractors for the quasi-geostrophic dynamical system on bounded and unbounded domains is discussed and also proved . Then are considered the global well-posedness and the asymptotic behavior of the 3D viscous PEs with random source in the form of an additive white noise in time. Two main theorems are stated and proved dealing with the existence and uniqueness of strong solutions for PEs and the existence of random attractors of the suitable stochastic dynamical system. The final section presents the similar analysis applied to the 3D viscous PEs with a random boundary.

Chapter 5, Stability and Instability Theory, contains a brief presentation of wave stability theory in geophysical fluid dynamics. Several kinds of hydrodynamic instabilities, e.g., neutral, linear, formal, and nonlinear instabilities are discussed. In Section 5.1, some results are recalled for the classical internal and internal inertial gravity waves problems. In particular, the stability and evolutionary properties of gravity waves are considered, based on the normal mode approach. The next section is devoted to selected results on the instability of Rossby waves, including the growth rate of the barotropic instable Rossby waves. The energy and normal mode methods are used. Another section reviews the nonlinear stability theory as applied to Rossby waves by means of an energy-Casimir method. Lastly, the Rayleigh-Bénard convection problem is analysed. The recent results on the linear and nonlinear stability of the stationary solution of the convective initial-boundary value problem, depending on the critical Rayleigh number, are formulated and proved.

This book is very well written, but it is not easy to read. For the relatively gentle introduction to the mathematical (geophysical) fluid dynamics, I strongly recommend the "Mathematics of Climate Modeling" by V.P. Dymnikov and A.N. Filatov (Birkhäuser, Boston 1997). The reader also needs some knowledge of special branches of mathematics: functional analysis, qualitative theory of differential equations, topology, stochastic analysis, etc. The book is highly recommended to researchers, graduate students included, who are interested in mathematical geophysics and dynamical systems theory methods.